Be sure that this examination has 13 pages including this cover

The University of British Columbia

Sessional Examinations - April 2013

Mathematics 101

Integral Calculus with Applications to Physical Sciences and Engineering

Closed book examination Time: 2.5 hours

Last Name: _____ **First Name:** _____

Student Number: _____ **Instructor's Name:** _____

Signature: _____ **Section Number:** _____

READ AND OBSERVE THE FOLLOWING EXAM RULES

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (i) speaking or communicating with other examination candidates, unless otherwise authorized;
   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (iii) purposely viewing the written papers of other examination candidates;
   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

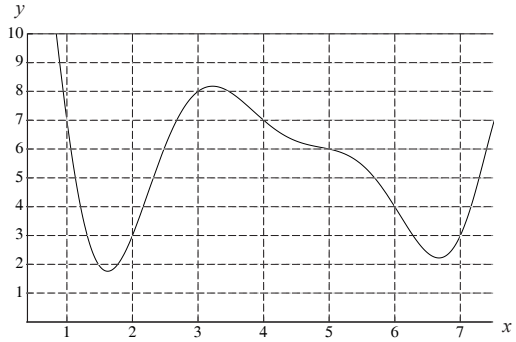
1		9
2		9
3		12
4		10
5		10
6		10
7		10
8		6
9		6
10		6
11		6
12		6
Total		100

Marks

Short-Answer Questions. Questions 1 – 3 are short-answer questions. Put your answer in the box provided. **Simplify your answer as much as possible.** Full marks will be awarded for a correct answer placed in the box. **Show your work,** for part marks.

[9] **1.** In each of the questions below, at most 1 mark will be awarded for an incorrect answer.

The graph below applies to both parts (a) and (b).



(a) [3] Use the Trapezoidal Rule, with $n = 4$, to estimate the area under the graph between $x = 2$ and $x = 6$. Remember to simplify your answer completely.

Answer

(b) [3] Use Simpson's Rule, with $n = 4$, to estimate the area under the graph between $x = 2$ and $x = 6$.

Answer

(c) [3] Evaluate $\displaystyle\int_1^2 \frac{dx}{x + x^2}$.

Answer

Continued on page 3

[9] **2.** In each of the questions below, at most 1 mark will be awarded for an incorrect answer.

(a) [3] Consider the sequence $\left\{(-1)^n \sin\left(\frac{1}{n}\right)\right\}$. State whether this sequence converges or diverges, and if it converges give its limit.

Answer

(b) [3] Find the sum of the series $\sum_{n=2}^{\infty} \frac{3 \cdot 4^{n+1}}{8 \cdot 5^n}$. Remember to simplify your answer completely.

Answer

(c) [3] The first two terms in the MacLaurin series for $x^2 \sin(x^3)$ are $ax^5 + bx^{11}$, where a and b are constants. Find the values of a and b.

Answer

Continued on page 4

[12] **3.** In each of the questions below, 1, 2, or 3 marks may be awarded for an incorrect answer. As in Questions 1 and 2, a correct, simplified answer generates full marks.

(a) [4] Evaluate $\int_1^3 (2x-1)e^{x^2-x}\,dx$.

Answer

(b) [4] A force of 10 N (newtons) is required to hold a spring stretched 5 cm beyond its natural length. How much work, in joules (J), is done in stretching the spring from its natural length to 50 cm beyond its natural length?

Answer

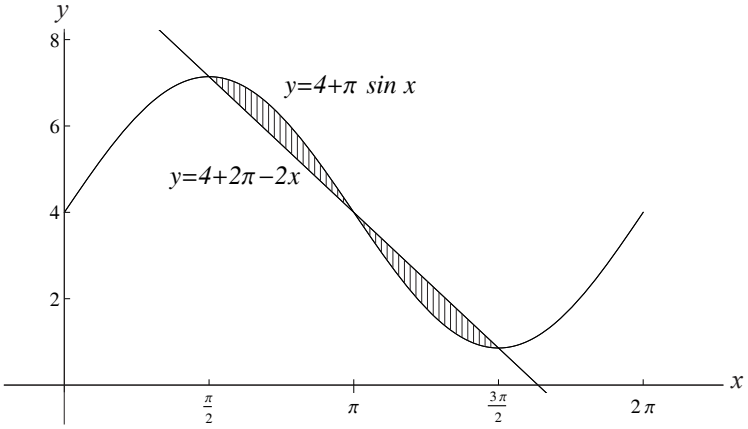
(c) [4] If $x\sin(\pi x) = \int_0^x f(t)\,dt$ where f is a continuous function, find $f(4)$.

Answer

Continued on page 5

Full-Solution Problems. In questions 4–12, justify your answers and **show all your work**. If a box is provided, write your final answer there. **Unless otherwise indicated, simplification of numerical answers is required in these questions.**

[10] **4.** The graph below shows the region between $y = 4 + \pi \sin x$ and $y = 4 + 2\pi - 2x$.



(a) [6] Find the area of this region.

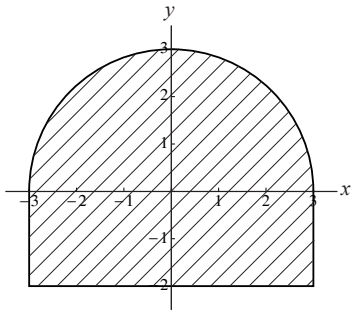
Answer

(b) [4] The region is rotated about the line $y = -1$. Express in terms of definite integrals the volume of the resulting solid. *Do not* evaluate the integrals. Also, it is not necessary to simplify them.

Continued on page 6

[10] **5.** (a) [5] Using a limit of right-endpoint Riemann sums, evaluate $\int_2^4 x^2\,dx$. No credit will be given for the use of antidifferentiation, but you may use it to check your answer. You may use the formulas $\sum_{i=1}^n i = n(n+1)/2$ and $\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$.

(b) [5] Find the centroid of the region below, which consists of a semicircle of radius 3 on top of a rectangle of width 6 and height 2.



Answer

Continued on page 7

[10] **6.** (a) [5] Evaluate $\int_0^1 \tan^{-1} x \; dx$.

Answer

(b) [5] Evaluate $\int \frac{2x-1}{x^2-2x+5} \; dx$.

Answer

Continued on page 8

[10] **7.** (a) [5] Show that $\int_0^{\pi/4} \cos^4\theta \, d\theta = (8+3\pi)/32$.

(b) [5] Evaluate $\int_{-1}^{1} \frac{dx}{(x^2+1)^3}$. You may use the result of part (a) above, whether or not you completed that problem.

Answer

Continued on page 9

[6] **8.** A tank contains 1000 L (litres) of pure water. A solution that contains 0.01 kg/L of sugar is poured into the tank at a rate of 20 L/min. The tank's contents are thoroughly mixed and drain out of the tank at the same rate. How much sugar is in the tank after one hour?

Answer

Continued on page 10

[6] **9.** Determine, with explanation, whether the following series converge or diverge.

(a) [3] $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots$

(b) [3] $\sum_{n=1}^{\infty} \frac{(2n+1)}{2^{2n+1}}$

Continued on page 11

[6] **10.** (a) [3] Find the power-series representation for $\int \frac{1}{1+x^3}\,dx$ centred at 0 (i.e. in powers of x).

(b) [3] The power series above is used to approximate $\int_0^{1/4} \frac{1}{1+x^3}\,dx$. How many terms are required to guarantee that the resulting approximation is within 10^{-5} of the exact value? Justify your answer.

Answer

Continued on page 12

[6] **11.** Consider the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{\sqrt{n}}$, where x is a real number. Find the interval of convergence of this series.

Answer

Continued on page 13

[6] **12.** (a) [2] Prove that $\displaystyle\int_2^\infty \frac{x+\sin x}{1+x^2}\,dx$ diverges.

(b) [2] Explain why you *cannot* conclude that $\displaystyle\sum_{n=2}^{\infty} \frac{n+\sin n}{1+n^2}$ diverges from part (a) and the Integral Test.

(c) [2] Determine, with explanation, whether $\displaystyle\sum_{n=2}^{\infty} \frac{n+\sin n}{1+n^2}$ converges or diverges.

The End