Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - April 2006

Mathematics 101

Integral Calculus

Closed book examination   Time: 2.5 hours

Print Name ——————————   **Signature** ————————————

Student Number——————————   **Instructor's Name** ————————————

                                   Section Number ————————————

Special Instructions:

No calculators, cell phones, notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than
is provided, use the back of the previous page.
Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1		33
2		17
3		20
4		12
5		10
6		4
7		4
Total		100

Marks

[33] **1.** **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for a correct answer placed in the box, but at most one mark will be given for an incorrect answer. Unless otherwise stated, simplify your answer as much as possible.

(a)   Find the average value of $\cos x$ on the interval $[0, \pi]$.

Answer

(b)   Evaluate $\displaystyle\int \cos^2 x \; dx$.

Answer

(c)   Evaluate $\displaystyle\int \left(\frac{1}{\sqrt{1-x^2}} + 1 - x^2 \right) dx$.

Answer

(d) Write down the Simpson's Rule approximation S_6 for $\int_2^8 \frac{1}{x^2}\,dx$. You may leave your answer expressed as a sum of fractions.

> Answer

(e) Calculate the volume of the solid obtained by rotating the region above the x-axis, below the curve $y = x + x^2$, and between the lines $x = 1$ and $x = 2$ about the x-axis.

> Answer

(f) Determine the general solution of the differential equation $y'' - y' - 2y = 0$.

> Answer

(g) Determine the general solution of the differential equation $y'' - 2y' + 2y = 0$.

> Answer

(h) A particle is moved along the x-axis by a force that measures $x^{3/2}$ N at a point x m from the origin. How much work is done in moving the particle from the origin to a point 10 m away from the origin?

Answer

(i) Determine whether $\displaystyle\int_1^\infty \frac{dx}{1+2x}$ converges or diverges. If it converges, evaluate it.

Answer

(j) Find the first three nonzero terms in the power series representation in powers of x (i.e. the Maclaurin series) for $\displaystyle\int_0^x t^2 e^{-t^2}\, dt$.

Answer

(k) Let $f(x) = \displaystyle\int_x^{x^2} \sqrt{t^4+1}\, dt$. Compute $f'(1)$.

Answer

Full-Solution Problems. In questions 2–7, justify your answers and **show all your work**. If a box is provided, write your final answer there. Unless otherwise indicated, simplification of answers is not required.

[17] **2.** (a) [5] Sketch the bounded region that lies between the curves $y = 2 - x^2$ and $y = |x|$, and show that the area of this region equals $7/3$.

(b) [3] Express the y-coordinate of the centroid (centre of mass) of the region in part (a) as a definite integral. *Do not evaluate this integral.*

Answer

(c)   [4] Determine the length of the curve $y = (2/3)x^{3/2}$, $0 \le x \le 3$. Evaluate and completely simplify your answer.

> Answer

(d)   [5] Express the volume of the solid obtained by rotating the bounded region that lies between the curves $y = (x-2)^2$ and $y = x$ about the vertical line $x = 5$. *Do not evaluate this integral.*

> Answer

[20] **3.** Evaluate the following integrals. Each part is worth 5 marks, but not all parts are of equal difficulty.

(a)

$$\int \frac{x^2}{(x^3 + 1)^{101}} \, dx$$

Answer

(b)

$$\int \frac{x^2 + 2}{x^3 + x} \, dx$$

Answer

(c)

$$\int \frac{x^2}{(100 - x^2)^{3/2}} \; dx$$

Answer

(d)

$$\int e^{2x} \sin 2x \; dx$$

Answer

[12] **4.** (a) [5] Solve the initial-value problem

$$\frac{dy}{dx} = y^4(x+1)^2, \ y(0) = -1.$$

Express your answer in the form $y = f(x)$, and simplify your answer completely.

Answer

(b) [7] Solve the initial-value problem $y'' + 2y' = \cos x, \ y(0) = 1, \ y'(0) = -1.$

Answer

[10] **5.** The vertical cross-sections of a bathtub perpendicular to its long axis have the shape shown below. They are squares of side length 50 cm surmounting a semicircle. Assuming the tub is full of water, find the hydrostatic force on one of its vertical ends (which also has the shape below). For the density of water, use the value 1000 kg/m^3, and for the acceleration due to gravity use the value 9.8 m/sec^2.

50 cm
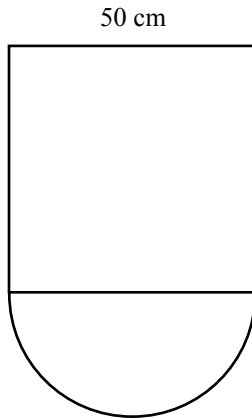
Answer

[4] **6.** X is a random variable with probability density function $f(x) = 2xe^{-x^2}$, where $x \geq 0$. Find the *median* of X.

> Answer

[4] **7.** Evaluate $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{(i+n)(i+2n)}$.

> Answer

The End