

The University of British Columbia

Final Examination - April 24, 2014

Mathematics 180

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____ Section Number _____ Instructor _____

Special Instructions:

No books, notes, or calculators are allowed.

Senate Policy: Conduct during examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1a,b,c,d,e		15
1f,g,h,i,j		15
1k,l,m,n		12
2		10
3		10
4		10
5		8
6		12
7		8
Total		100

[42] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Sketch the graph of an example of a function $f(x)$ that satisfies the following conditions:

$$\lim_{x \rightarrow 0^-} f(x) = 3, \quad \lim_{x \rightarrow 0^+} f(x) = -1, \quad f(0) = 2.$$

Answer:

(b) Evaluate the limit $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$.

Answer:

(c) If $8x \leq f(x) \leq x^2 + 16$ for $x \geq 0$, find $\lim_{x \rightarrow 4} f(x)$.

Answer:

(d) Is the following function

$$f(x) = \begin{cases} e^x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

continuous at $a = 1$?

Answer:

(e) Evaluate the limit $\lim_{x \rightarrow \infty} \frac{(3x^2 - 1)^2}{(x - 2)^2(x^2 - 3)}$.

Answer:

(f) If a ball is thrown into the air with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

Answer:

(g) Find a function $f(x)$ and a number c such that $f'(c) = \lim_{x \rightarrow 4} \frac{3^x - 81}{x - 4}$.

Answer:

(h) If $f(x) = \frac{e^x}{g(x)}$, where $g(0) = 2$ and $g'(0) = -1$, find $f'(0)$.

Answer:

(i) If $g(x) = f(3f(x))$, where $f(1) = \frac{1}{3}$ and $f'(1) = -3$, find $g'(1)$.

Answer:

(j) If a function $y = f(x)$ is defined implicitly by an equation $y + x^2y^5 = 27$, find $\frac{dy}{dx}$.

Answer:

(k) If $y = x^{\cos x}$, find $\frac{dy}{dx}$.

Answer:

(l) Use a linear approximation to estimate $\sqrt{81.1}$.

Answer:

(m) If $f(0) = 8$ and $f'(x) \geq 3$ for $0 \leq x \leq 5$, how small can $f(5)$ possibly be?

Answer:

(n) Use Newton's method with an initial approximation of $x_1 = 1$ to find x_2 , where x_2 is the second approximation to the root of the equation $x^5 - x - 1 = 0$.

Answer:

Full-Solution Problems. In questions 2 – 7, justify your answers and show all your work. Unless otherwise indicated, simplification of numerical answers is required in these questions.

[10] **2.** Using the definition of derivative, compute $f'(x)$ if $f(x) = \frac{x}{3x - 5}$. No marks will be given for the use of differential rules, but you may use them to check your answer.

[10] **3.** When a cake is removed from an oven, the temperature of the cake is 190°F . The cake is left to cool at room temperature, which is 72°F . After 25 min the temperature of the cake is 150°F . When will the cake have cooled to 100°F ?

[10] 4. Car A is traveling east at 65 mi/h and Car B is traveling south at 50 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when Car A is 0.4 mi and Car B is 0.3 mi from the intersection?

[8] **5.** A right circular cylinder is inscribed in a sphere of radius 3. Find the largest possible volume of such a cylinder.

[12] 6. A function $y = f(x)$ defined on $(-\infty, \infty)$ satisfies the following conditions:

$$f(0) = 0, \quad f'(x) = e^{-x^2}(1 - 2x^2), \quad \lim_{x \rightarrow \pm\infty} f(x) = 0.$$

(a) Find the critical numbers of $y = f(x)$.

(b) Determine the intervals where $f(x)$ is increasing, and the intervals where $f(x)$ is decreasing.

(c) Determine the intervals where $f(x)$ is concave up, and the intervals where $f(x)$ is concave down.

(d) Sketch the graph of $y = f(x)$ and indicate the inflection points on your graph.

[8] 7. If $f(x)$ is a continuous function with domain $[1, 2]$ and range in $[1, 4]$, prove that there exists a number c in $[1, 2]$ such that $f(c) = c^2$.