Marks

[42]   **1.**   **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a)   Evaluate $\displaystyle\lim_{t\to-2} \frac{2t^2+t-6}{t^2-4}$ or determine that the limit does not exist.

> Answer

(b)   Evaluate $\displaystyle\lim_{x\to1} \frac{\sqrt{x+3}-2}{x-1}$ or determine that the limit does not exist.

> Answer

(c)   Evaluate $\displaystyle\lim_{x\to\infty} \frac{2x}{\sqrt{9x^2+x-1}}$ or determine that the limit does not exist.

> Answer

(d) Suppose f and g are continuous functions such that $g(3) = 2$ and $\lim_{x \to 3} \left(x f(x) + g(x) \right) = 1$. Find $f(3)$.

> Answer

(e) Find an equation of the tangent line to the curve $y = x^{3.5} - e^{3.5}$ at the point $(e, 0)$.

> Answer

(f) Find the derivative of $f(x) = \dfrac{x}{x^2 - 1}$. Please simplify your answer.

> Answer

(g) A vertical cylindrical tank with radius 3m is being filled with water at a rate of $5\text{m}^3/\text{min}$. How fast is the height of the water increasing?

> Answer

(h) Find the slope of the tangent line to the curve $y = f(x)$ at the point $(0, 2)$, where the function $y = f(x)$ satisfies the equation: $x^2 + xy + y^2 = 4$.

Answer

(i) Find the derivative of $y = \sin^{-1}(3x + 1)$. [Note: Another notation for $\sin^{-1}$ is arcsin.]

Answer

(j) Find $f'(x)$, where $f(x) = x^{(x^2)}$.

Answer

(k) Find $\dfrac{dy}{dt}$, where $y = e^{t\cos(2t)}$.

Answer

(l)   If $f(-1) = 9$ and $f'(x) \geq 3$ for $-1 \leq x \leq 2$, how small can $f(2)$ possible be?

Answer

(m)   Use Newton's Method to find the second approximation x_2 to $\sqrt[6]{2}$, starting with the initial approximation $x_1 = 1$.

Answer

(n)   Find a function $f(x)$ and a number a such that

$$2 f'(a) = \lim_{h \to 0} \frac{2 \cos\left(\dfrac{\pi}{3} + h\right) - 1}{h}.$$

Answer

Full-Solution Problems. In questions 2 8, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required unless explicitly requested.

[8]   **2.**   A wealthy man was found murdered in his home. The police arrived on the scene at 10:00 P.M. The temperature of the body at 10:00 P.M. was $33°C$ and one hour later it was $31°C$. The temperature of the room in which the body was found was $21°C$ and normal body temperature is $37°C$. Assume that the body cools after death according to Newton's Law of Cooling.

      (a)   How many hours before the police arrived on the scene did the murder occur?

      (b)   Did the murder occur before 9:00 P.M.? Please justify your answer.

to (a)

Answer

to (b)

Answer

[8]　**3.**　A particle moves in a straight line according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in meters. The acceleration function $a(t)$ is given by $a(t) = 12t - 30$, the velocity after $2s$ is 0, and $f(0) = 0$.

　　(a)　Find the position function $s = f(t)$.

　　(b)　Find the total distance traveled by the particle during the first three seconds.

[7]   **4.**   Let $T_2(x)$ be the second degree Taylor polynomial about $a = 8$ for $f(x) = \sqrt[3]{x}$.

(a)   Find $T_2(x)$ and simplify your answer.

(b)   Is $T_2(8.1)$ larger than $\sqrt[3]{8.1}$? Please justify your answer.

[12]   **5.**   Let $y = f(x) = 6x^{\frac{1}{5}} + x^{\frac{6}{5}}$.

     (a)   Find the critical numbers of $y = f(x)$.

     (b)   Find the interval(s) of decrease.

     (c)   Find the second derivative of $y = f(x)$.

Question 5 continued

(d)   Find the intervals on which $y = f(x)$ is concave up.

(e)   Sketch the graph of $y = f(x)$ and indicate the inflection points on your graph. Please also indicate the point(s) on your graph at which the graph of $y = f(x)$ has a vertical tangent line.

[8]　**6.**　If 24m^2 of material is available to make a rectangular storage container with an open top, and if the length of its base is twice the width, find the largest possible volume of the rectangular storage container. Please justify that your answer gives indeed the largest possible volume.

[8]   **7.**   Let

$$f(x) = \begin{cases} 4x - \dfrac{\sin^2(2x-2)}{3x-3} - 1 & \text{if } x < 1 \\[2ex] 2x + 1 & \text{if } x \geq 1. \end{cases}$$

Use the definition of derivative to find $f'(1)$ or show that $f'(1)$ does not exist.

> Answer

[7] **8.** If $y = f(x)$ is a continuous function with domain $[0,1]$ and range in $[3,5]$, prove that the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.

The End

Be sure that this examination has 13 pages including this cover

The University of British Columbia

Sessional Examinations - December 2011

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination   Time: 2.5 hours

Surname(s): _____   Given Name(s): _____

Student Number:_____   Instructor's Name: _____

Signature:_____   Section Number: _____

Special Instructions:

No books, notes, or calculators are allowed. Unless otherwise specified, answers that are numerical constants may be left in "calculator-ready" form, where calculator means basic scientific calculator. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

1		42
2		8
3		8
4		7
5		12
6		8
7		8
8		7
Total		100