Marks

[42] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate $\lim_{x\to 1} \dfrac{x^2 - x}{x^2 - 1}$ or determine that this limit does not exist.

Answer

(b) Evaluate $\lim_{x\to\infty} \dfrac{x^3 + 5x}{2x^3 - x^2 + 4}$ or determine that this limit does not exist.

Answer

(c) Find the derivative of $\dfrac{x}{3 + e^x}$.

Answer

Continued on page 3

(d) Find the derivative of $t^3 \cos t$.

Answer

(e) Find the derivative of $e^{\sqrt{x}}$.

Answer

(f) Find $f'(x)$, if $f(x) = \arctan(x^3)$. [*Note:* Another notation for arctan is $\tan^{-1}$.]

Answer

(g) If $x^2 + xy - y^2 = 4$, find dy/dx in terms of x and y.

Answer

Continued on page 4

(h) If $f(x) = (\cos x)^x$, find $f'(x)$.

Answer

(i) Use a linear approximation to approximate $\sqrt{100.2}$.

Answer

(j) Estimate the size of the error made in the linear approximation above. In other words, find an upper bound for the absolute value of the difference between $\sqrt{100.2}$ and the answer to item (i) above.

Answer

(k) If $f(x) = x\cos(x^2)$, compute $f^{(9)}(0)$. *Hint:* Use Maclaurin series.

Answer

Continued on page 5

(l) Find the absolute minimum *value* of $f(x) = \sin^{-1}(x/2)$ on the interval $[-2, -1]$. [*Note:* Another notation for $\sin^{-1}$ is arcsin.]

Answer

(m) Newton's Method is used to approximate a solution of the equation $\sin x = 1 - x$, starting with the initial approximation $x_1 = \pi/2$. Find x_2.

Answer

(n) Given that $f'(x) = 2x - (3/x^4)$, $x > 0$, and $f(1) = 3$, find $f(x)$.

Answer

Continued on page 6

Full-Solution Problems. In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required unless explicitly requested.

[10] **2.** A freshly brewed cup of coffee initially has temperature 95°C in a room that has fixed temperature 20°C. When the coffee temperature is 70°C, it is decreasing at a rate of 1°C per minute. When does this occur? Assume that the temperature of the coffee in the cup satisfies Newton's Law of Cooling.

Answer

Continued on page 7

[10] **3.** Gravel is being dumped from a conveyor belt at a rate of 2 m^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 3 m high? *Hint:* The volume of a right circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.

Answer

Continued on page 8

[12] **4.** Let $f(x) = \dfrac{x^2 + 12}{x - 2}$.

(a) (1 mark) Find the domain of $f(x)$.

(b) (4 marks) Determine intervals where $f(x)$ is increasing or decreasing and the x- and y-coordinates of all local maxima or minima (if any).

(c) (2 marks) Determine intervals where $f(x)$ is concave upwards or downwards, and the x-coordinates of all inflection points (if any). You may use, without verifying it, the formula $f''(x) = 32/(x-2)^3$.

Question 4 continues on the next page...

Question 4 continued

(d) (3 marks) Find and verify the equations of any asymptotes (horizontal, vertical or slant).

(e) (4 marks) Sketch the graph of $y = f(x)$, showing the features given in items (a) to (d) above and giving the (x, y) coordinates for all points occurring above and also all x-intercepts (if any).

Continued on page 10

[10] **5.** A cylindrical can without a top is made to contain 27π cm^3 of liquid. Determine (with justification) the dimensions of the can that minimize the area of the metal used to make the can. *Hint:* The metal used consists of a circle (the bottom of the can), and a rectangle (the sides of the can).

Answer

Continued on page 11

[4] **6.** Use the *definition of the derivative* to determine $f'(x)$, where

$$f(x) = \frac{1}{x+5}.$$

No credit will be given for using derivative formulas.

Continued on page 12

[4] **7.** The function $f(x)$ is defined by

$$f(x) = \begin{cases} ax^2 + bx + c & \text{if } x < 0, \\ 2 & \text{if } x = 0, \\ 2 + x^2 \cos(x^{-1}) & \text{if } x > 0. \end{cases}$$

Determine all values of the constants a, b, c such that f continuous at 0, or determine that no such values exist.

Continued on page 13

[8] **8.** Let $\mathcal{L}$ be the tangent line to the curve $y = 1 + e^x$ at a point $P(a, b)$, where the point of tangency P is chosen so that tangent line $\mathcal{L}$ passes through the origin $O(0, 0)$.

(a) (3 marks) Find an equation that is satisfied by a, the x-coordinate of the point P.

(b) (5 marks) Prove that the equation in item (a) has exactly one real solution.

The End

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The University of British Columbia

Sessional Examinations - December 2008

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination | Time: 2.5 hours

Surname(s): ____________________ **Given Name(s):** ____________________

Student Number: ____________________ **Instructor's Name:** ____________________

Signature: ____________________ **Section Number:** ____________________

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2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

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(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		42
2		10
3		10
4		12
5		10
6		4
7		4
8		8
Total		100