Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - December 2006

Mathematics 100/180

Differential Calculus with Applications to Physical Sciences and Engineering

Closed book examination   Time: 2.5 hours

Last Name: _____   **First Name:** _____

Student Number: _____   **Instructor's Name:** _____

Signature: _____   **Section Number:** _____

Rules governing examinations

1		42
2		10
3		12
4		16
5		10
6		10
Total		100

Marks

[42] **1. Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. Full marks will be given for correct answers placed in the box, but at most 1 mark will be given for incorrect answers. Unless otherwise stated, it is not necessary to simplify your answers in this question.

(a) Evaluate $\lim\limits_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 16}$

> Answer

(b) Evaluate $\lim\limits_{x \to \infty} \dfrac{x^3 + 5x}{2x^3 - x^2 + 4}$

> Answer

(c) Find the value of the constant a for which $\lim\limits_{x \to -2} \dfrac{x^2 + ax + 3}{x^2 + x - 2}$ exists.

> Answer

(d) Find the values of the constants a and b for which

$$f(x) = \begin{cases} x^2 \text{ if } x \leq 2 \\ ax + b \text{ if } x > 2 \end{cases}$$

is differentiable everywhere.

Answer

(e) Find the derivative of $e^{x \cos x}$.

Answer

(f) If $x^2 y^2 + x \sin y = 4$, find dy/dx.

Answer

(g) Let $f(x) = x^3 \ln x$. Find the 4th derivative of $f(x)$, i.e. $f^{(4)}(x)$.

Answer

(h) Find the derivative of $\sin^{-1}(e^{2x})$. Note: Another notation for $\sin^{-1}$ is arcsin.

> Answer

(i) Find the derivative of $(\tan x)^x$.

> Answer

(j) Using a suitable linear approximation, estimate $(8.06)^{2/3}$. Give your answer as a fraction in which both the numerator and denominator are integers.

> Answer

(k) Write down the first three nonzero terms in the Maclaurin series for $x \sin(-2x)$.

> Answer

(l) A function $f(x)$ has third derivative equal to $10/(1 - x)$. The second-degree Taylor polynomial $T_2(x)$ at 0 is used to approximate $f(0.1)$. Find the upper bound for the error given by Taylor's Inequality, i.e. the upper bound for $|f(0.1) - T_2(0.1)|$.

Answer

(m) Using Newton's Method, starting with $x_1 = 1$, find the approximation x_3 to the root of the equation $x^3 - x^2 - 1 = 0$.

Answer

(n) A particle moves in a straight line so that its velocity at time t is $v(t) = t^3$. If its position at time t is $s(t)$ and $s(1) = 2$, find $s(2)$.

Answer

Name:

Full-Solution Problems. In questions 2–6, justify your answers and **show all your work**. If a box is provided, write your final answer there. Simplification of answers is not required.

[10] **2.** A poster is to have a total area of 180 cm^2 with 1-cm margins at the bottom and sides and a 2-cm margin at the top. What dimensions will give the largest printed area?

Answer

[12]　**3.**　Two cities, Growth and Decay, have populations that are respectively increasing and decreasing, at (different) rates that are proportional to the respective current population. Growth's population is now 3 million and was 2 million 10 years ago, and Decay's population is now 5 million and was 7 million 10 years ago.

(a)　(6 marks) How many years from now will Growth's population equal 4 million?

Answer

(b)　(6 marks) How many years from now will Growth and Decay have the same population?

Answer

[16] **4.** The function $f(x)$ is defined by

$$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ \dfrac{x^2 + 3}{3(x+1)} & \text{if } x \geq 0 \end{cases}$$

(a) (2 marks) Explain why $f(x)$ is continuous everywhere.

(b) Determine all of the following if they are present:

(i) (4 marks) x-coordinates of local maxima and minima, intervals where $f(x)$ is increasing or decreasing;

(ii) (4 marks) intervals where $f(x)$ is concave upwards or downwards;

Question 4(b) continued

   (iii) (3 marks) equations of any asymptotes (horizontal, vertical, or slant).

   (c)   (3 marks) Sketch the graph of $y = f(x)$, giving the (x, y) coordinates for all points of
         interest above.

[10] **5.** A trough is 10 m long and its ends have the shape of equilateral triangles (i.e. all three sides have equal length) that are 2 m across, as shown in the diagram below. If the trough is being filled with water at the rate of 12 m³/min, how fast is the water level rising when the water is 60 cm deep?

Answer
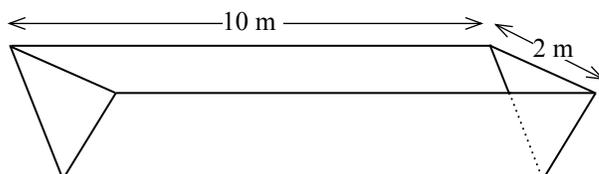
[10] **6.**   (a)   (6 marks) There are two distinct straight lines that pass through the point $(1, -3)$ and are tangent to the curve $y = x^2$. Find equations for these two lines.

> Answer

(b)   (4 marks) Evaluate
$$\lim_{x \to 0} x^{1/101} \sin(x^{-100})$$

or explain why this limit does not exist. Give complete justification for your answer.

The End