SUMMER 2016
NSERC USRA REPORT
STATISTICS OF MULTIPLICATIVE GROUPS

STUDENT: BEN CHANG
SUPERVISOR: GREG MARTIN

1. Brief of Study

We consider some statistics of multiplicative groups $(\mathbb{Z}/n\mathbb{Z})^\times$ for $n \geq 3$. The fact that these multiplicative groups can be represented as direct sums of cyclic groups allows us to study some intrinsic arithmetic properties about n by examining their cyclic factors.

The canonical structures of these multiplicative groups in terms of primary and invariant factor decomposition inspire certain number theoretic questions we can answer, such as the number of these cyclic factors in the decomposition.

In particular, the order of magnitude of the number of positive integers $n \leq x$ such that $(\mathbb{Z}/n\mathbb{Z})^\times$ has no cyclic factor of order 2 is about $x/\sqrt{\log x}$; and a theorem of Erdos and Kac suggests that the number of cyclic factors of $(\mathbb{Z}/n\mathbb{Z})^\times$ with n uniformly sampled in the range $[1, x]$ is asymptotically normally distributed.

2. Scope of Project

We plan to produce an expository paper outlining the canonical structures of multiplicative groups and how these relate to questions in number theory; and a research paper on applying known results in number theory to estimate the number of multiplicative groups with the least factor greater than 2 and to determine the limiting distribution concerning the number of primary factors.

3. Work Performed

I started by collecting results I learned in courses concerning number theory and group theory to build up tools we need to analyze the canonical structures of multiplicative groups.

Under the guide of my supervisor, I verified statements about the arithmetic properties of n which describe structures of the multiplicative group $(\mathbb{Z}/n\mathbb{Z})^\times$, such as the number of factors in the decomposition.

After studying various texts in analytic number theory and papers by Erdos, Pomerance, my supervisor, and others, I learned answers to some well-known number theoretic questions relating to the canonical structures of multiplicative groups, and techniques for estimating summatary functions involving certain arithmetical functions.

Under the guide of my supervisor, I worked out the details in proving the estimates for the number of n with least primary/invariant factor greater than 2 by applying Selberg Delange method and other analysis techniques.

Using a paper by Erdos and Pomerance, "On the Normal Number of Prime Factors of $\phi(n)$", as a template, I showed that the number of primary factors, just like for the invariant factors, has a limiting normal distribution.

4. Results

We produced an exposition on the canonical structures of multiplicative groups and their connections to number theory.

We also produced new results on the estimates of certain arithmetical functions involving counting primes.