NSERC USRA Report
Smooth Particle Hydrodynamics

Adam Ng

September 12, 2011

The project is orientated to extending a finite-element method to simulate incompressible-Newtonian Fluid flows in channel. It is, to the furthest, considering cases for 2-dimensional geometry; and the fluid dynamic is governed by momentum conservation, mass conservation, and density conservation. Explicitly, we have Navier-Stoke Equation

$$\rho\frac{d\mathbf{V}}{dt} = \mu\nabla^2 - \nabla\mathbf{P} + \vec{g}$$

with incompressibility, $\nabla\mathbf{V} = 0$; and mass conservation is achieved via holding the total number of particles constant.

This formulation requires well defined boundary conditions; and solvers to those systems are, so far, computationally expensive. The existing SPH-algorithm solves the dynamic in a Projection-Correction Scheme extrapolate the $\hat{u^{k+1}}$, the velocity-projection, without pressure term and later correct it via solving the p^{k+1} as an explicit function of density. Moreover, since field variables is approximated by discrete elements, we have a general solver

$$< A_h(\vec{r}) >= \sum_j \mathbf{V}_j A(\vec{r_j})\mathbf{W}(\vec{r} - \vec{r_j}, h)$$

where $\mathbf{V}$ denotes volume of particle j, $\mathbf{W}$ denotes weight function satisfying $\lim_{h\to 0} = \delta$, and respectively $\vec{r}$ denotes location.

With this finite-element time-step differencing method, we did not regard the incompressibility at projection step; hence upon correction, we restore the change in density with

$$\frac{1}{\rho_o}\frac{\rho_o - \rho}{dt} + \nabla\mathbf{V^c} = 0$$

and taking the divergence of NS, we have

$$\nabla(frac1rho\nabla\mathbf{P}) = \frac{\rho_o - \rho}{\rho_o\Delta t^2}$$

This algorithm does indeed provide convergent result for Couette flow and Poiseuille flow; however, when we attempt computing geometry involving expansion/contraction, the solution degenerates.

Main hypothetical issue with the old code is the lacking of formalized boundary condition. For the successful runs, we, by simplicity of the geometry, does not have velocity change on the normal component of the fluid about the walls – which suffices a Neumann boundary condition for pressure. To by pass this issue, we, at cost of computation step, seek a better appropriate solver to solve the Poisson-Pressure Equation

$$\frac{\nabla \hat{u}^{k+1}}{\Delta t} = \frac{1}{\rho}\nabla^2 \phi^{k+1}$$

, where $\phi^{k+1} = p^{k+1} - pk + \mu \nabla \hat{u}^{k+1}$,

and the projection steps involves all terms in NS equation.

Unsurprisingly, this formulation suffices a well-defined Neumann-Pressure boundary condition as we combine the two terms into the projection equation, duplicating a time-step discrete NS equation, and take divergence of it.

Unfortunately, this was yet completed before the end of my summer placement due to large amount of coding effort required. The project is now continued as a undergraduate placement of mine and further result is to be seen