Summer 2010 - NSERC USRA Report: The Distribution of the Eigenvalues of Covers of a Graph

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This summer I worked in spectral graph theory under the supervision of Dr. Lior Silberman. The goal of the project was to numerically look at the distribution of the largest new eigenvalue of random covers of certain base graphs. Then, based on the data, to predict the limit distribution as the size of the cover is taken to infinity. Emphasis was put on distributions which appear in Random Matrix Theory, such as the Tracy Widom GUE, and GOE distributions.

Experimental Approach:

A program was written in MATLAB to generate a large number of covers of a fixed degree for a chosen base graph, and then find the eigenvalues. For each experiment one million graphs were created and the distribution was compared to the Tracy Widom distributions. The maximum difference between cumulative distributions was looked at, as well as the skewness. All distributions were normalized to have mean 0 and standard deviation 1.

Results:

Covers of Regular Graphs:

Several different base graphs were used, in particular $K5$, $K4$, the Peterson graph, and the vertex with two loops. For each of these base graphs the maximum CDF difference between the data and the Tracy Widom GOE distribution was converging to 0. The skewness was also very close to 0.293, the skewness of the Tracy Widom GOE. This strongly suggests that the limit distribution for the largest new eigenvalue of covers of d-regular graphs is in fact the Tracy Widom GOE distribution. Interestingly convergence depended on the covers being simple (this only affects covers of the vertex with two loops).

Covers of Irregular Graphs:

Several different base graphs were used, in particular K4 with one edge taken away. The numerical evidence showed with very little doubt that the distribu-

tion of the largest new eigenvalue of covers of irregular graphs does not converge to any of the Tracy Widom distributions.

Abelian Covers of Graphs:

First a d-regular base graph was taken, either $K5$ or $K10$. A large cover was then generated, and each entry above the diagonal in the adjacency matrix was multiplied by a random complex number on the unit circle. The lower triangle was chosen so that the matrix would be Hermitian. The result was that the distribution of the largest eigenvalue followed the Tracy Widom GUE distribution. The CDF difference was converging to 0, and the skewness was very close to 0.224. (which is the skewness of the GUE)

Below is the maximum CDF difference between the distribution of the largest new eigenvalue of K5 and the Tracy Widom GOE distribution plotted against the size of the graph (base size * degree of cover)