NSERC USRA Report: The Spectra Of Random Graphs

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This summer I worked with Dr. Lior Silberman and we looked at the spectra of random graphs. The project began as a series of numerical experiments, most of which found data in agreement with known theorems and results. As we moved to irregular graphs, the observations made led us to several conjectures, and the work then became analytic. Our main result is:

Theorem 1. *Let G is a finite connected undirected graph with minimum degree at least 2, let λ is the largest eigenvalue of the adjacency matrix A, and let γ is the largest eigenvalue of the Hashimoto directed edge matrix. Then $\frac{\delta_G-1}{\Delta_G-1} \leq \lambda-\gamma \leq \frac{\Delta_G-1}{\delta_G-1}$ where Δ_G and δ_G are the maximum and minimum degree of any vertex in the graph.*

In fact, there is a conjecture that $\lambda-\gamma \geq 1$, and from the work done on Theorem 1 (as well as numerical results) it would be very surprising if false. It was also found numerically that the distribution for λ_1, where λ_1 largest eigenvalue of the cover which is not an eigenvalue of the base graph G, for high degree covers of G can be closely approximated by the Tracy Widom Distribution ($\beta = 2$). In particular we looked at each cumulative distribution to make the comparison, and found that the Tracy Widom curve was a much better fit then the Gaussian distribution (as an example). Similar numerical experiments were done looking at the exact value of λ_1, which led me to the following conjecture:

Conjecture 2. *If $\rho(\widetilde{G})$ is the spectral radius of the universal cover of a graph G with minimum degree 2, $|V| = n$, and γ is defined as above, then $2\sqrt{\Lambda} \leq 2\sqrt{\gamma} \leq \rho(\widetilde{G})$ where*

$$\Lambda = \prod_{i=1}^{n}(d_i-1)^{d_i/\sum_{j=1}^{n} d_j},$$

and $d_1, d_2, ..., d_n$ is a degree sequence of the graph.

It should be noted that in a paper by Shlomo Hoory he proved that $2\sqrt{\Lambda} \leq \rho(\widetilde{G})$ under the conditions of the above conjecture.

As it is there is much work to be done on these problems as well as several other conjectures (that I am not as certain about as the one mentioned) which I hope to continue during term. In conclusion, i found my research experience this summer invaluable, as working with Dr. Silberman as a USRA student taught me more then any course could have.