NSERC RESEARCH REPORT

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Polynomials are fundamental objects in mathematics, and the study of the zero set of multi-variable polynomials is an extremely rich and diverse area at the interface of classical analysis and algebraic geometry. In classical harmonic analysis, the structures of these zero sets play a significant role in problems involving integrals with polynomial singularities, and in decay estimates for scalar oscillatory integrals and integral operators. While such problems have been completely settled where the ambient polynomial is a function of two variables (see, for example, [2], and [3]) many of them remain open in dimensions three and higher, largely due to the dramatic increase in complexity of the structure of the zero set. On the other hand, the method of resolution of singularities in algebraic geometry (a vast body of work originating from a result of Hironaka in 1964) aims to harness these complexities in a methodical manner. A fundamental challenge in this area has been to adapt the known schemes for resolving the singularities of multidimensional polynomials to tackle the analytical problems mentioned above. During the summer of 2008, in collaboration with Dr. Malabika Pramanik, I undertook the study of convergence criterion for a certain class of singular integrals. In particular, we examined integrals of the form

$$\int_{\Omega} |f|^{-\delta} \, \mathrm{dV}, \tag{1}$$

where $f : \mathbb{R}^3 \to \mathbb{R}$ is an analytic function satisfying $f(0) = 0$ and $\Omega \subset \mathbb{R}^n$ is a neighbourhood of the origin. Our goal was to classify the critical integrability index, defined as the number

$$\tilde{\delta} = \sup\{\delta : \text{ the integral in (1) converges}\}.$$

This problem has been examined and solved in the case that $f : \mathbb{R}^2 \to \mathbb{R}$ (see [2]), however, in 3 and higher dimensions it has remained open. In our work, Dr. Pramanik and I were able to develop a geometric classification the critical integrability index for arbitrary analytic functions $f : \mathbb{R}^3 \to \mathbb{R}$. Our work uses in a fundamental way the Jung-Abhyankhar Theorem of algebraic geometry, as well as an explicit resolution of singularities algorithm in the spirit of that in [1]. Our approach is to resolve the singularities of a certain polynomial in two variables which implies that the zero variety of the function f becomes controllable in an analytic sense. In this way, our approach seems to suggest that an inductive argument might develop a similar classification of the critical integrability index in general dimensions. However, there are a number of technical difficulties which need to be overcome for this approach to be productive in dimensions higher than 3.

References.

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