Equality of Schur Q-functions

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September 2, 2008

In learning about quasisymmetric functions I was naturally led to investigate equality of Schur Q-functions. What follows is three of my most significant results produced during my summer research, which will result in a journal article joint with Steph van Willigenburg.

1 Definitions

- Let α° and α^t represent, respectively, the rotation by 180 degrees and transpose of the composition α.
- Let $\mathfrak{r}_\alpha$ denote the skew Schur Q-function whose shifted diagram corresponds to the ribbon α.
- Let $\mathfrak{s}_D$ denote the skew Schur Q-function whose shifted diagram corresponds to the *ordinary* skew diagram D.
- Let $\bullet$ represent the bullet operation: $\alpha \bullet \beta$ means that we take $|\alpha|$ copies of β, alternatively transpose them, and then glue them according to α.

2 Three of my results

Theorem 2.1 *For compositions α, β and skew diagram D, if $\mathfrak{r}_\alpha = \mathfrak{r}_\beta$ then $\mathfrak{s}_{\alpha \bullet D} = \mathfrak{s}_{\beta \bullet D}$.*

Theorem 2.2 *The skew diagram $\alpha_1 \bullet \cdots \bullet \alpha_m \bullet D$ has the same ordinary skew Schur Q-function as those skew diagrams $\beta_1 \bullet \cdots \bullet \beta_m \bullet E$ where $\beta_i = \{\alpha_i, {\alpha_i}^t, {\alpha_i}^\circ, ({\alpha_i}^t)^\circ\}$ for $1 \leq i \leq m$, and $E = \{D, D^t, D^\circ, (D^t)^\circ\}$.*

Theorem 2.3 *(i) For $|\alpha|$ odd, the ribbon Schur Q-function $\mathfrak{r}_\alpha$ is irreducible considered as an element of $\mathbb{Z}[q_1, q_3, \ldots]$.*
(ii) For $|\alpha|$ even, there are infinitely many examples in which $\mathfrak{r}_\alpha$ is irreducible and infinitely many examples in which $\mathfrak{r}_\alpha$ is reducible.

*Research supported by NSERC