

# The Numerical Solution of a Finger-Like Fracture in Viscosity and Toughness Regimes

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## 1 Summary

This project concerns the solution of an approximate model for a planar hydraulic fracture which is constrained to propagate within a horizontal strip bounded by two parallel lines. Such fracture geometries can be caused for example by large jumps in the ambient geological confining stresses across the two parallel interfaces which cause a stress barrier to vertical fracture propagation. In this situation the fracture can only propagate horizontally, which makes the problem amenable to approximation by one dimensional models. Indeed, the simplest approximation due to Perkins and Kern [2] makes the assumption that the fracture width is varying sufficiently slowly in the horizontal direction that a state of plane strain exists for any vertical cross section, so that the nonlocal elasticity equation reduces to the simple local relation:

$$p(x) = \frac{E'}{4H} w_o(x) \quad (1)$$

where  $E' = E/(1 - \nu^2)$  is the plane-strain Young's modulus. Since the fracture is not propagating in the vertical direction the vertical fluid velocity will be negligible compared to the horizontal velocity. Thus Adachi and Peirce [1] assumed that there is no variation in the fluid pressure in the vertical direction and that the associated fracture width field has an elliptical cross section. By separating the variables in the width field it is possible to reduce the two-dimensional integral equation into a one-dimensional integral equation, which is able to represent the nonlocal effects of the elasticity operator. The one dimensional integral equation can be shown to reduce to the local equation (1) in regions away from the tip in the limit of large aspect ratio fractures. Moreover, the asymptotic analysis in [1] established the asymptotic action of the nonlocal operator for a power law test function as the pressure sample point approaches the propagating tip of the fracture.

In this project we combine the one dimensional integral equation with the lubrication equation and the appropriate boundary and propagation conditions into a one dimensional model of a propagating two-dimensional finger-like hydraulic fracture. The integral equation is discretized by assuming that the fracture width field is piecewise constant, while the finite volume method is used to discretize the lubrication equation.

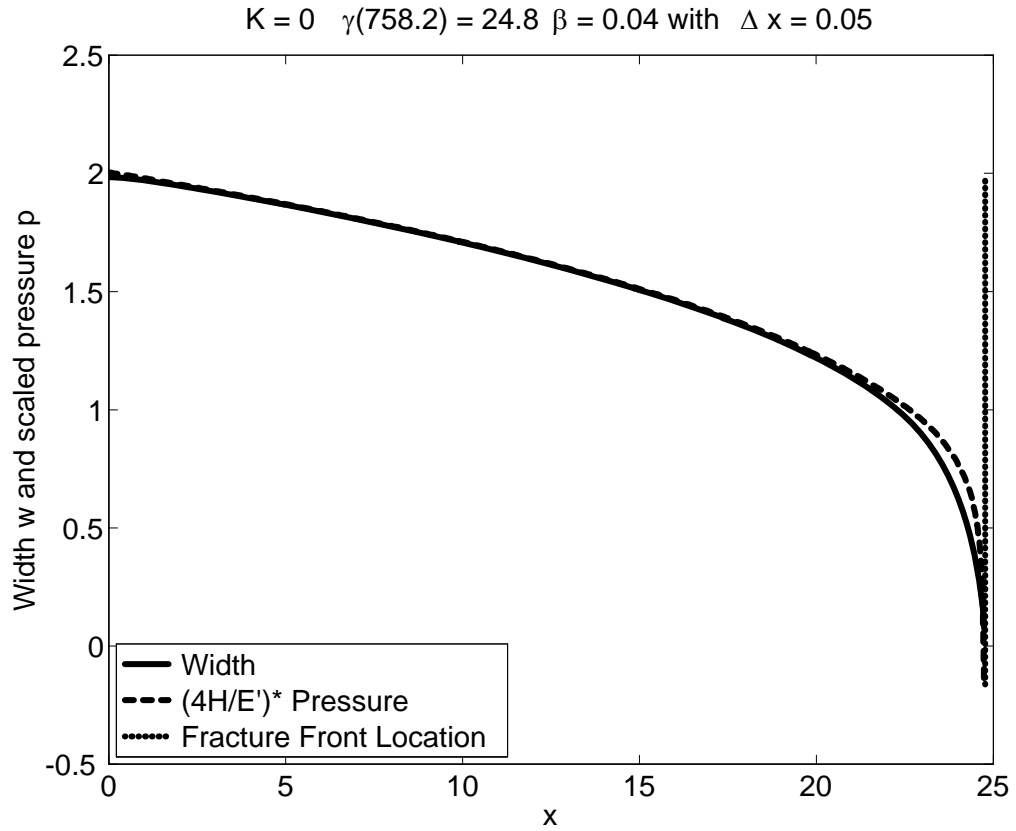


Figure 1: Fracture width and scaled pressure for the zero toughness case  $K = 0$

The local tip solution, which can be obtained by assuming a power law behaviour and a dominant balance argument, is used to locate the free boundary of the propagating fracture. We briefly show some results below. In these plots the parameter  $\gamma$  represents the dimensionless length of the fracture, and  $\beta = H/l$  represents the height to length ratio of the fracture.

In figure 1 we plot the fracture width and scaled pressure for a finger-like fracture propagating in a zero toughness regime. The agreement between the width and the scaled pressure is close for points away from the tip, which demonstrates the region of applicability of the approximation in equation (1).

In figure 2 we plot the log of the fracture width against distance from the tip for the zero toughness case  $K = 0$ . A number of regimes of behaviour can be identified. The  $2/3$  power law represents the tip asymptote for a viscous fracture, the  $1/3$  power represents the similarity solution associated with the local pressure width relation given in equation (1), while the  $1/4$  power is associated with the well bore.

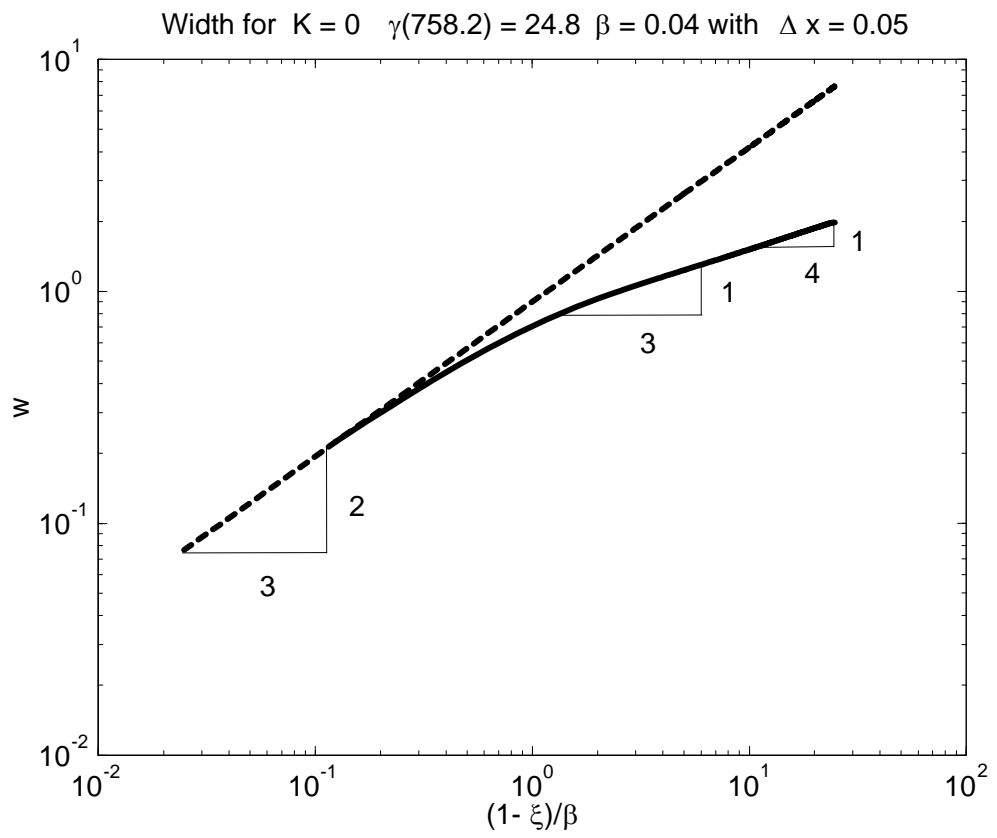


Figure 2: Log-log plot of the fracture width against the distance from the tip.

Width for  $K = 1$   $\gamma(747) = 23.5$   $\beta = 0.042$  with  $\Delta x = 0.05$

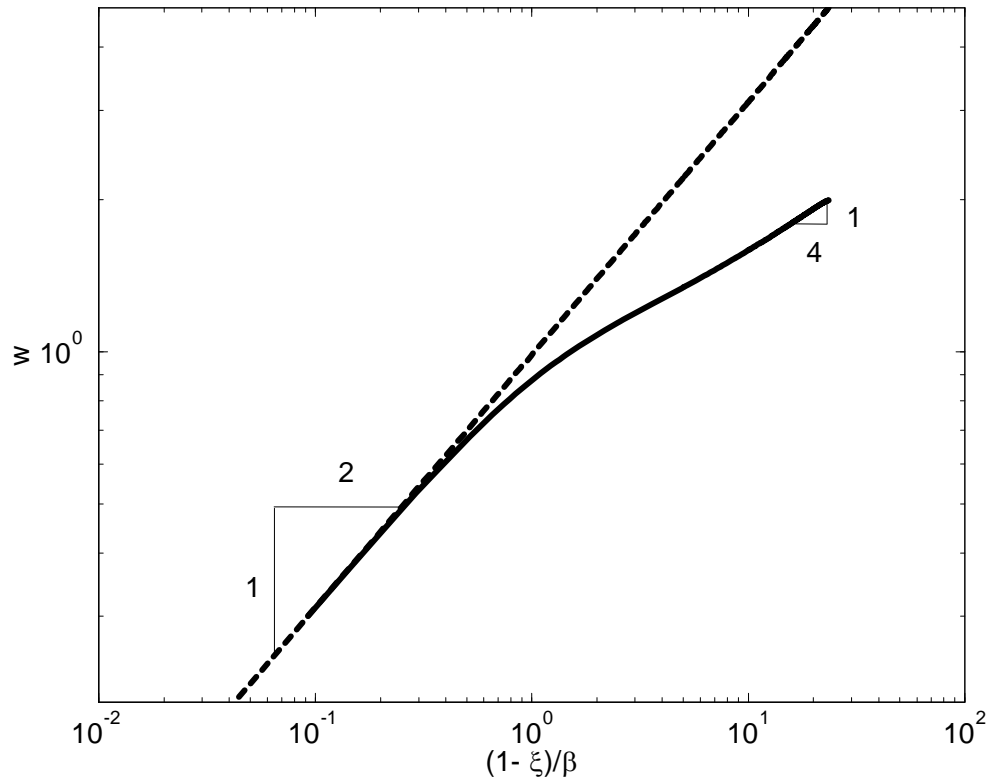


Figure 3: Log-log plot of the fracture width against distance from the tip for the case  $K = 1$ .

In figure 3 we plot we plot the log of the fracture width against the log of the distance from the tip for the case in which the dimensionless toughness  $K = 1$ . In this case the characteristic  $1/2$  power law tip behaviour for a non-zero toughness can be clearly observed.

In figure 4 we plot we plot the log of the fracture width against the log of the distance from the tip for the case in which the dimensionless toughness  $K = 5$ . In this case the characteristic  $1/2$  power law tip behaviour for a non-zero toughness can be clearly observed while the width (and the pressure) become close to constant away from the fracture tip.

The numerical scheme was able to identify a number of different propagation regimes as the dimensionless toughness is varied. This analysis clearly demonstrates the limitations of the classical PKN models which cannot include the effect of the pressure singularities associated with either viscous or toughness dominated fracture propagation. The PKN model is expected to be mainly applicable in situations in which

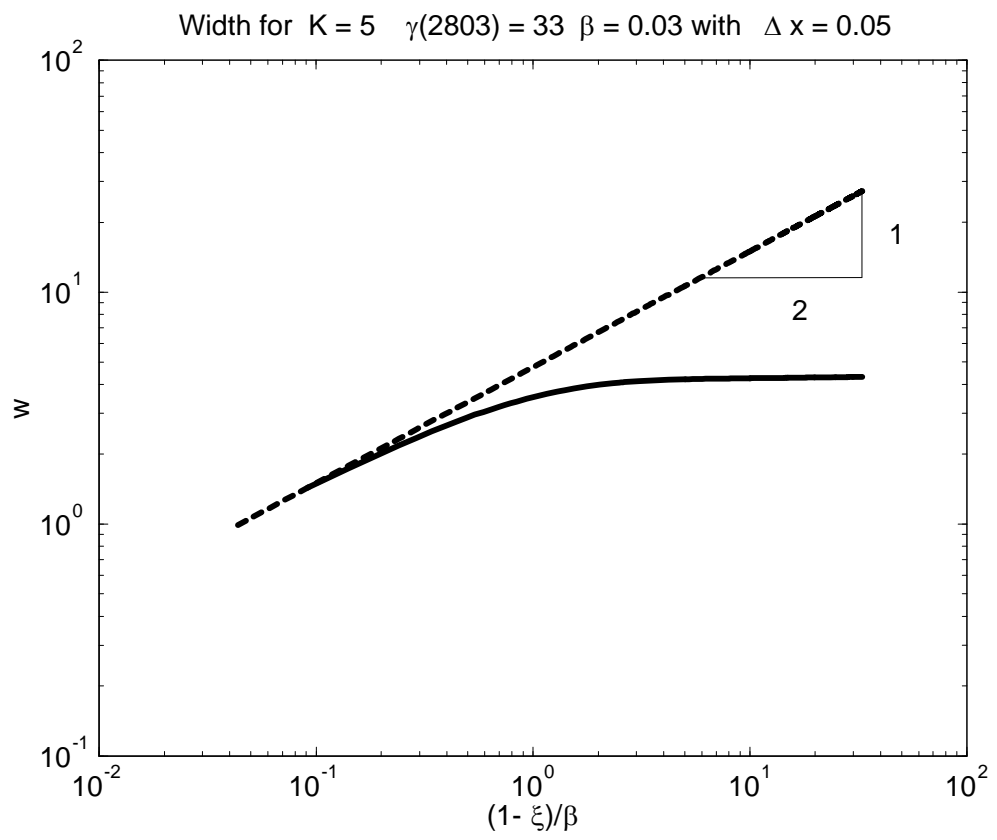


Figure 4:

toughness is negligible while it will yield erroneous results for problems in which the toughness dominates the propagation of the fracture.

## References

- [1] J.L.Adachi and A.P. Peirce, Asymptotic Analysis of an Elasticity Equation for a finger-like Hydraulic Fracture, *J. Elasticity*. **90** (2008) 43–69.
- [2] T. Perkins and L. Kern, Widths of hydraulic fractures. *J. Pet. Tech.* **222** (1961) 937-949.