Decomposition of Principle Series Representations via Elementary Methods NSERC USRA

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Abstract

Let G be a split reductive p-adic group. We have sufficient conditions for $Ind_B^G\lambda$ to be decomposable as a direct sum $(Ind_P^G\chi\otimes 1)\oplus(Ind_P^G\chi\otimes st)$. Here, λ is a character of a maximal torus T contained in some borel subgroup B. χ is a character of the levi L of a rank one parabolic subgroup P containing B. 1 denotes the trivial representation and st denotes the steinberg representation. These sufficient conditions can be manipulated so that some conditions about the real part of λ realized as a vector in the root space of the root system associated with G imply that the aforementioned decomposition of $Ind_B^G\lambda$ holds. The conditions can be expressed as a system of equations and inequalities in a finite dimensional$\mathbb{R}$- vector space. We implemented code in CoCalc that would solve the system of equations and inequalities and thus give specific characters where the aforementioned decomposition is valid, and found a solution that uniformly holds for the classical groups A_n, B_n, C_n, and D_n, as well as the five exceptional groups G_2, F_4, E_6, E_7, and E_8.

1 Introduction

We are interested in characters λ of a maximal torus T in a borel B, such that

$$Ind_B^G\lambda = (Ind_P^G\chi \otimes 1) \oplus (Ind_P^G\chi \otimes st),$$

for some character χ of the Levi of P where P is a parabolic of rank one associated to the simple root α. We have that this holds if we have the following conditions on $\lambda_o = Re(\lambda)$ for some w in the Weyl Group that does not commute with the reflection across the simple root:

1. $<\lambda_o, \alpha^v>= 1$,
2. $ws_\alpha(\lambda_o) = s_\alpha(\lambda_o)$,
3. For every $\beta \neq \alpha$ that is a positive root, $<\lambda_o + \frac{\alpha}{2},\beta^v>\leq 0$, and
4. For every β that is a simple root, $<s_\alpha(\lambda_o), \beta^v>\leq 0$.

2 Methods

We created a function in CoCalc that given a Chevalley group G, a simple root α, a weyl element w, returns the system of equations corresponding to conditions 1 - 4, using a basis of fundamental weights. Then, we created another function that given a particular group G, ran the previous function, looping over all simple roots and looping over all weyl elements not commuting with s_α. The code was run for many small classical groups, as well as the three smallest exceptional groups. We were able to use this to classify the embeddings of rank 2 root systems within the four infinite families of classical groups.

The code was too time consuming to run for the bigger exceptional groups, and another algorithm was written to obtain $s_\alpha(\lambda_o)$ instead. The new function, given a group G, a simple root α, and a set of simple roots not containing α, S, gave a system of equations that gave $s_\alpha(\lambda_o)$ where λ_o satisfied the conditions above. This function was also looped over the simple roots and the subsets of simple roots. The advantage of this function was that it ran much faster.

3 Results

We were able to show that there is a uniform solution of the form $s_\alpha(-\omega_\alpha)$. Here, ω_α is the unique character where $< \omega_\alpha, \alpha^v >= 1$ and $< \omega_\alpha, \beta^v >= 0$ for $\beta \neq \alpha$.

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