

NSERC USRA Report

Novel Approximation Schemes to model Hydraulic Fracture

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This summer I worked on a project involving approximation schemes for modelling hydraulic fractures, under the supervision of Professor Anthony Peirce.

The first part of the summer was spent researching and trying methods for speeding up an existing numerical scheme developed by Professor Peirce, with a focus on parallelism through frameworks like OpenMP (CPU) and CUDA (GPU). The principle complication was that the initial scheme was written in Matlab and that OpenMP and CUDA can be accessed directly only through programming languages like C, C++, and occasionally also Fortran. This problem was circumvented using Matlab's MEX files, which provides the missing link to C, C++, or Fortran. Unfortunately, these methods ended up speeding up the scheme only marginally.

Afterwards, I undertook to solve the integral differential equation

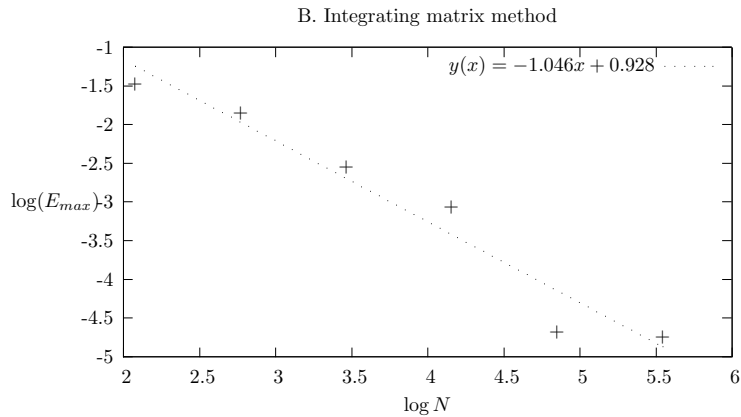
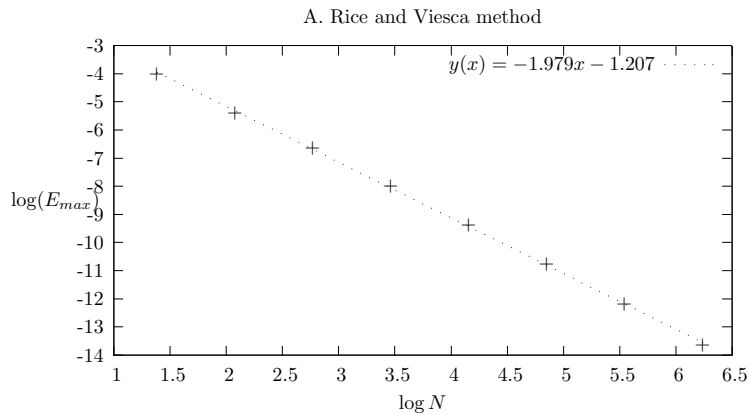
$$\Pi(x) = \frac{1}{\pi} \int_{-1}^1 \frac{w'(s)}{x-s} ds \text{ for } -1 < x < 1.$$

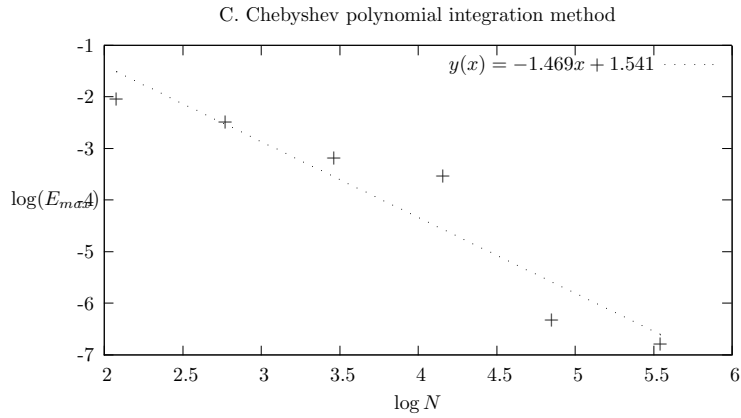
This equation is used to model the crack width w as a function of the pressure Π . Various spectral methods were used to approximate solutions.

- The simplest approximation for the equation was using piecewise constant basis functions. This served as a reference point for the other schemes. Almost exact solutions were obtained by using thousands of approximation points. These “exact” solutions were compared to the results obtained from other schemes when using relatively fewer node points.
- Viesca and Rice [1] discuss a change of variables that allows us to use the Gauss-Chebyshev quadrature approximation coupled with an explicit integration scheme. This method obtained a second order convergence.
- Based off the Gauss-Chebyshev quadrature again, this scheme made use of Chebyshev spectral differentiation in hopes of obtaining high convergence.

- Once a spectral sampling of w' was extracted from the quadrature, the Chebyshev matrix was inverted (using an extra initial condition) and multiplied with w' to extract w . This method had first order convergence.
- Another analogous approximation was used integrating Chebyshev polynomials symbolically using identities of the form of $\int T_n dx = \frac{n}{n^2-1}T_{n+1} - \frac{x}{n-1}T_n$. The order of convergence improved slightly to 1.5

The log-log plots of these methods are included below for reference.





Another method involving the Gauss-Jacobi quadrature was not finished. The motivation behind this quadrature would have been to fit the fracture tip asymptotic behaviour.

References

- [1] Viesca, R. C., and J. R. Rice (2012), Nucleation of slip-weakening rupture instability in landslides by localized increase of pore pressure, *J. Geophys. Res.*, 117, B03104, doi:10.1029/2011JB008866