Mathematical modules in Science One

Science One is a highly selective first year program at UBC for students eager to explore various disciplines in science and push themselves in their studies. Science One aims to motivate students to learn about different fields of study in an integrated fashion and to apply their knowledge from one discipline to understanding another. In light of this teaching philosophy, our goal this summer was to modify the math section of Science One to reflect better the interdisciplinary goals of the program. We chose to break up the Science One math curriculum into several modules, each consisting of one or more mathematical topics.

These modules are each centred around a question from other subjects taught in Science One (biology, chemistry, or physics) which can be approached and answered from a mathematical perspective. These modules were drafted through the following process.

First, the regular first year curriculum and supplementary materials were divided into several sections based on related topics. Next, we brainstormed specific questions in other sciences that might reflect one or more topics in the sections. After defining the theme question for the module, we searched for previous studies that have addressed, or attempted to address, the question. We then drafted the module by stating the question, providing any background information, and exploring the relevant topics in mathematics. These concepts will help the students approach the problem from a mathematical point of view and formulate their answer to the question step by step. Often, the question was too complex for the scope of first year mathematics curriculum, and the problem would need to be reduced to a simpler model. In these cases, any underlying assumptions were carefully defined, and the assumptions were justified in the context of the original problem.

In the end, we were able to split each semester into 4 modules. These were written in the form of lecture notes for the instructor. The final product is a set of 8 folders. Each folder includes the lecture-style notes, the background knowledge to present the question, and any research material we have found surrounding the topic that might be useful. These modules apply concepts from different fields of study taught in Science One to reflect the interdisciplinary nature of the science one program.

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The following are the 8 modules, with a brief abstract and topics covered:

1. Population dynamics - Population dynamics for the number of fish in a fishery is analyzed using limits and derivatives. A recursive model for the fish population (Ricker model) is presented, and the characteristics for the steady state of the fish population (represented by a fixed point in the Ricker model) is investigated by the use of first derivative at the fixed point.
2. Ray optics/Satellite dish - The reflective properties of circular and parabolic reflectors are examined. The slopes of the reflective surfaces are computed using different differentiating techniques (explicit differentiation, implicit differentiation, chain rule). An alternative approach using vector analysis of an imaginary particle tracing the circular/parabolic trajectories is introduced as a supplemental topic.
3. Biofuel - The rate of growth of volume (biomass) of a given poplar tree is determined from its growth pattern using related rates, and the optimal harvesting strategy is determined based on this growth pattern in order to optimize the rate of biomass production.
4. Morse potential/Harmonic oscillators - Potential energy between two covalently-bonded atoms is expressed as a function of the distance between them, and the critical points, inflection points and first/second derivatives are determined in order to sketch the graph of the function. The chemical and physical significances of the shape of the graph for different types of bonds are discussed.
5. Air Particulates - The size distribution of air pollutant particles are found based on the size density function for the particles. This module introduces the Fundamental Theorem of Calculus and the significance of the area under the curve for the particle size density function.
6. Applications of integration - The theoretical amount of wind energy produced from a wind mill in a given period of time is estimated from the area swept by the blades and the wind speed frequency distribution (Weibull distribution). This module motivates the use of several integration techniques, e.g. integration by substitution. Other applications of integration (e.g. volume of rotation, Gini coefficient) are also discussed.
7. Particle in a cylindrical box - Schrodinger's equation in cylindrical coordinates is expressed as Bessel differential equation with a change of variables, and the power

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series solution to the differential equation (Bessel functions of the first kind) is determined. The concept of uniform convergence is discussed to justify the term-by-term differentiation of the power series.

8. Reaction kinetics and *lac operon* - The expressions for the rates of reactions with zeroth-order, first-order, and Michaelis-Mentel kinetics are derived using differential equations. The gene regulation by the *lac* operon is briefly discussed to motivate an introduction to systems of differential equations.

The lectures will be scheduled so that these topics are introduced to the students at approximately the same time that they are covered in the other subjects. In giving students multiple perspectives from different disciplines at the same time, the connections between different fields of science will be clearer with ample opportunity for the students to ask questions. By introducing interdisciplinary curriculum for Science One mathematics, other subjects may be encouraged to also relate their topics with other sciences. From there, it is imperative that the students are encouraged to find their own connections by actively finding other examples of interdisciplinary problems that have not yet been mentioned. Such thinking would encourage students to consider the course material in a new light, as well as allow them to communicate their creative ideas with their peers.