

NSERC USRA Report - Curvature Induced Forces and Torques on Spheroidal Particles on Fluid Interfaces

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When a solid particle, such as a dust mote, is placed on a fluid interface, such as a soap film or the surface of a pond, it causes a distortion to the interface. For the case of static fluids, the interface must satisfy a differential equation called the Young-Laplace equation

$$\Delta P = -\sigma \nabla \cdot \vec{n}, \quad (1)$$

where ΔP is the pressure difference across the interface, σ is the surface tension or energy per unit area of the interface, and \vec{n} is the unit normal to the surface. In order to solve for the interfacial shape, we need to impose boundary conditions on the problem. Fortunately, such a condition is imposed on the line where the particle meets the interface. Depending on the three materials involved, the normal vector to the interface and the normal vector to the particle's surface must have a fixed angle between them. This is referred to as the contact angle condition. Since the interface distortion is effected by the particle, varying particle position or orientation may lead to changes in interfacial shape and hence interfacial energy. The goal of this project is to study the changes in interfacial energy caused by moving the particle in order to understand the forces acting upon it.

In many cases, however, it is extremely difficult to solve the Young-Laplace equation and determine interfacial energy. For this reason, we use the finite element program Surface Evolver to model the systems being studied. By approximating the surface as a finite union of triangles, and evolving towards the lowest energy, this program allows us to solve the Young-Laplace equation for more difficult interfaces.

The early stages of the project focus on validating our methodology for several previously researched systems. In particular, the results of Alois Würger [2] which indicated a net force on spherical particles on minimal surfaces towards the direction of minimal principal curvature magnitude. The distortion around a nearly spherical prolate spheroid was also studied in order to make predictions regarding the orientation behaviour of such a particle on the catenoid. It was observed that the distortion was similar to that of rod-like particles with

complimentary contact angles. As experimental studies have shown [1], cylindrical particles of contact angles greater than $\frac{3\pi}{2}$ tend to orient horizontally across interfaces similar to the catenoid, it was expected that spheroids would also display orientation preferences. However, unlike the work with cylinders, we examined contact angles above and below $\frac{\pi}{2}$ which causes inversion of the distortion. Thus, a contact angle dependence not discussed in the previous work was hypothesized.

From these basic cases, we switched our focus to the more interesting problem of a spheroid on a catenoid. In particular, we focused on the effect of contact angle on particle orientation. As predicted, a contact angle dependence did emerge with particles having contact angles less than $\frac{\pi}{2}$ orienting across the surface while other particles oriented in the direction of increasing Z . Preliminary studies were also done on the forces acting to move the particle along the fluid interface. As with the sphere, the early results indicate that particles move towards the point of minimal principal curvature magnitude.

References

- [1] Marcello Cavallaro, Lorenzo Botto, Eric P. Lewandowski, Marisa Wang, and Kathleen J. Stebe. Curvature-driven capillary migration and assembly of rod-like particles. *Proceedings of the National Academy of Sciences*, 108, 2011.
- [2] Alois Würger. Curvature-induced capillary interaction of spherical particles at a liquid interface. *Phys. Rev. E*, 74:041402, Oct 2006.