

## Differential Equations Qualifying Examination Syllabus

### Elementary linear algebra

Systems of linear equations and their solutions, matrices, Gaussian elimination, row-reduced forms. Invertible matrices, determinants, Vandermonde matrices. Eigenvalues and eigenvectors.

### Vector spaces

Subspaces and quotient spaces, bases, dimension. Linear transformations, change of basis and similarity of matrices, rank and nullity. Inner product spaces, orthogonality, orthogonal complements, orthonormal bases, Gram–Schmidt. Adjoints of linear transformations, dual spaces.

### Diagonalizability of matrices

Relationship to eigenvectors. Symmetric matrices, diagonalization of symmetric matrices and normal operators, quadratic forms. Minimal and characteristic polynomials, Jordan canonical form. Exponentiation of matrices and applications to differential equations.

### Scalar ordinary differential equations

First order equations. The phase-line and stability of equilibria for autonomous first order equations. Second order differential equations with constant coefficients. Method of reduction of order. Variation of parameters. Sturm–Liouville theory. Power series solutions around ordinary points and regular singular points. Applications.

### Linear vector ordinary differential equations

Solution of constant coefficient linear systems using eigen-analysis. Wronskian, method of undetermined coefficients, resonance. Applications.

### Nonlinear vector ordinary differential equations

Autonomous systems. The phase-plane. Elementary notions of stability, classification of critical points.

### General ordinary differential equations

Conversion of higher order ODE problems to first-order systems. Laplace transform solutions. Nonlinear conservative systems.

### Partial differential equations

Qualitative behaviour of solutions to the heat equation, wave equation, and Laplace's equation. Separation of variables solutions. Laplace and Fourier transform solutions to the heat equation.

(continued)

**Suggested References** (note: not all topics in these sources are necessary for the qualifying examinations—refer to the above list of topics)

Boyce and Diprima, Differential Equations

Borrelli and Coleman, Differential Equations

Friedberg, Insel, and Spence, Linear Algebra

Haberman, Applied Partial Differential Equations

Hoffman and Kunze, Linear Algebra

Strang, Linear Algebra and its Applications

Strauss, Partial Differential Equations