Analysis Qualifying Examination Syllabus

Differentiation in $\mathbf{R}^n \to \mathbf{R}^m$

Partial derivatives, equality of mixed partials, directional derivatives, gradient, Jacobian, chain rule, divergence, curl, Taylor expansions, maxima and minima, Lagrange multipliers.

Vector calculus

Parametrization of curves and surfaces, outward unit normal direction to surfaces, conservative vector fields, gradient/potential fields.

Integration in $\mathbf{R}^n \to \mathbf{R}^m$

Riemann integration. Fubini theorems, line integrals, surface integrals, Gauss's theorem, Green's theorem, Stokes's theorem, changes of variables (e.g., cartesian to polar), improper integrals, differentiation under the integral sign.

Fundamental real analysis

Sequences, series, convergence tests, limits, continuity, epsilon-delta definitions, uniform continuity, differentiability, inverse and implicit function theorems. Cauchy–Schwarz inequality, mean value theorem (derivative and integral forms), contraction mappings and Banach fixed point theorem.

Analysis on functions

Sequences and series of functions, uniform convergence, term-by-term differentiation and integration of series of functions, Weierstrass approximation, elementary Fourier series.

Basic properties of analytic functions

The complex number system, the Cauchy–Riemann equations, analytic functions, elementary functions (e.g., rational functions, exponential function, logarithm function, trigonometric functions, roots) and their basic properties, analyticity of limit functions. Power series and Laurent series, isolation of zeros, classification of isolated singularities (including singularity at ∞), analytic continuation.

Harmonic functions and bounds on analytic functions

Basic properties of harmonic functions in the plane, connection with analytic functions, harmonic conjugates, mean value property. Maximum principle, Schwarz's lemma, Liouville's theorem.

Complex integration

Cauchy's integral formula, Cauchy's theorem, Morera's theorem, the residue theorem, evaluation of definite integrals. The argument principle and the logarithmic derivative, Rouche's theorem. Fourier transform and Fourier inversion.

(continued)

Complex mappings

Conformality of analytic functions, mapping properties of linear fractional transformations, conformal mappings (for example, to and from the unit disk and the upper half-plane), branch cuts.

Suggested References (note: not all topics in these sources are necessary for the qualifying examinations—refer to the above list of topics)

Ahlfors, Complex Analysis
Brown and Churchill, Complex Variables and Applications
Conway, Functions of One Complex Variable
Edwards, Advanced Calculus of Several Variables
Fisher, Complex Variables
Folland, Real Analysis
Gamelin, Complex Analysis
Gamelin and Greene, Introduction to Topology
Greenspan and Benney, Calculus: An introduction to applied mathematics
Marsden, Elementary Classical Analysis
Marsden and Hoffman, Basic Complex Analysis
Rosenlicht, Introduction to Analysis
Rudin, Principles of Mathematical Analysis
Rudin, Real and Complex Analysis
Saff and Snider, Complex Analysis for Math, Science, and Engineering