Algebra Qualifying Examination Syllabus

Elementary linear algebra

Systems of linear equations and their solutions, matrices, Gaussian elimination, row-reduced forms. Invertible matrices, determinants, Vandermonde matrices. Eigenvalues and eigenvectors.

Vector spaces

Subspaces and quotient spaces, bases, dimension. Linear transformations, change of basis and similarity of matrices, rank and nullity. Inner product spaces, Cauchy–Schwarz inequality, orthogonality, orthogonal complements, orthonormal bases, Gram–Schmidt. Adjoints of linear transformations, dual spaces.

Diagonalizability of matrices

Relationship to eigenvectors. Symmetric matrices, diagonalization of symmetric matrices and normal operators, quadratic forms. Minimal and characteristic polynomials, Cayley–Hamilton theorem, Jordan canonical form. Exponentiation of matrices.

Groups

Group actions, permutations, symmetric groups and alternating groups. Subgroups, cosets, Lagrange's theorem. Group homomorphisms, normal subgroups, quotient groups, solvable groups. Dihedral groups. Classification of finitely generated abelian groups. Sylow theorems.

Rings

Ideals, prime ideals, maximal ideals. Ring homomorphisms, kernels, quotient rings. Principal ideal domains, unique factorization, Euclidean domains. Unipotent and nilpotent elements. Rings of polynomials, factorization of polynomials, Gauss's lemma, Eisenstein's criterion.

Fields

Field extensions, degrees, finite fields. Splitting fields, Galois extensions, the Galois group, the fundamental theorem of Galois theory, solvability in radicals.

Suggested References (note: not all topics in these sources are necessary for the qualifying examinations—refer to the above list of topics)

Dummit and Foote, Abstract Algebra
Friedberg, Insel, and Spence, Linear Algebra
Gallian, Contemporary Abstract Algebra
Halmos, Finite-dimensional Vector Spaces
Hoffman and Kunze, Linear Algebra
Rotman, Introduction to the Theory of Groups
Stewart, Galois Theory